



**Q1:** Start from Schrodinger equation in three-dimension

$$\left[ \frac{-\hbar^2}{2m} \nabla^2 + V(r) \right] \Psi(\vec{r}) = E\Psi(\vec{r})$$

and use  $\nabla^2$  in spherical coordinate, where

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

and the variable separation

$$\psi = \frac{u(r)}{r} Y_{\ell m}(\theta, \phi)$$

to rewrite the Schrodinger equation to be looks like

$$\frac{d^2 u_{\ell}}{dr^2} + \frac{2m}{\hbar^2} \left[ E - V(r) - \frac{\ell(\ell + 1)}{2mr^2} \right] u_{\ell}(r) = 0$$

where  $\ell(\ell + 1)$  is the solution value of  $Y_{\ell m}(\theta, \phi)$

**Q2:** Determine the nucleus in  $A = 135$  isobars that have the lowest mass (largest binding energy) using the mass parabola. (*Hint*) differentiate the mass equation  $(Zm_Z + Nm_N - BE)$  with respect to  $Z$  with constant  $A$  and determine the extreme value of  $Z$  then recycle it to a closest integer, then determine the nucleus. (*Answer*  $Z=56$ ).